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EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS WITH NON-SEPARATED TYPE INTEGRAL BOUNDARY CONDITIONS*

Dedicated to Professor Peter D. Lax on the occasion of his 85th birthday

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Abstract In this paper, we study a boundary value problem of nonlinear fractional differential equations of order q $(1 < q \leq 2)$ with non-separated integral boundary conditions. Some new existence and uniqueness results are obtained by using some standard fixed point theorems and Leray-Schauder degree theory. Some illustrative examples are also presented. We extend previous results even in the integer case $q = 2$.

Key words fractional differential equations; non-separated integral boundary conditions; contraction principle; Krasnoselskii's fixed point theorem; Leray-Schauder degree

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1 Introduction

Integral boundary conditions have various applications in applied fields such as blood flow problems, chemical engineering, thermoelasticity, underground water flow, population dynamics, etc. For a detailed description of the integral boundary conditions, we refer the reader, for example, to the recent papers [2, 15].

Fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes. These characteristics of the fractional derivatives make the fractional-order models more realistic and practical than the classical integer-order models. As a matter of fact, fractional differential equations arise in many engineering and scientific disciplines such as physics, chemistry, biology, economics, control theory,

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