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Influence of Certain Permutable Subgroups on Finite Smooth Groups

A. M. ELKHOLY
Department of Mathematics, Beni-Suef University, Faculty of Science 62511, Beni-Suef, Egypt
E-mail: aelkholy9@yahoo.com

A. A. HELIEL
Department of Mathematics, King Abdulaziz University, Faculty of Science 80203, Jeddah 21589, Saudi Arabia
and
Department of Mathematics, Beni-Suef University, Faculty of Science 62511, Beni-Suef, Egypt
E-mail: heliel9@yahoo.com

Abstract A subgroup H of a finite group G is said to be permutable in G if it permutes with every subgroup of G. In this paper, we determine the finite groups which have a permutable subgroup of prime order and whose maximal subgroups are totally (generalized) smooth groups.

Keywords Permutable subgroups, smooth groups, subgroup lattices

MR(2000) Subject Classification 20D10, 20D20, 20E15, 20E16

1 Introduction

All groups considered in this paper will be finite. We use the standard notions and notations given by Schmidt in [1]. In addition, for a fixed group G, n will denote the maximal length of L(G) and the set of distinct primes dividing |G| will be denoted by $\pi(G)$.

A subgroup of a group G is permutable in G if it permutes with every subgroup of G. A subgroup M of a group G is called modular in G if it is modular in the subgroup lattice, L(G), of G. It is well known that the subgroup M of a finite group G is permutable if and only if M is modular and subnormal in G (see [1, p. 201, Theorem 5.1.1]). A maximal chain $0 = a_0 < a_1 < \cdots < a_n = I$ in a lattice L with least element 0 and greatest element I is called smooth if $[a_{i+j}/a_j] = [a_i/0]$ for all i, j N such that $i + j \leq n$. A group G is called smooth if its subgroup lattice L(G) has a smooth chain. Finite smooth groups have been studied by Schmidt [2, 3]. A lattice L is called totally smooth if all maximal chains of elements of L are smooth. A group G is said to be totally smooth if its subgroup lattice L(G) is totally smooth. A group G is generalized smooth if [G/H] is totally smooth for every subgroup H of prime order. Totally smooth groups and generalized smooth groups have been studied in [4] and [5], respectively.

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